## MATH-434 Numerical Linear Algebra

**Credit Hours:** 3-0

**Prerequisite**: MATH-221 Linear Algebra

**Course Objectives:** Numerical linear algebra is of great practical importance in scientific computation and is used in mathematics, natural sciences, computer science and social science. Even nonlinear problems usually involve linear algebra in their solution. The focus of the course is to explore applications in industry including direct implications for internet applications.

**Core Contents:** Review of matrix operations, QR factorization and least squares, conditioning and stability, numerical solutions of systems of linear equations including direct methods, error analysis, structured matrices, iterative methods, least squares and algorithms for eigenvalues and eigenvectors.

**Detailed Course Contents**: Matrix computations: Matrix-vector multiplication, Orthogonalvectors and matrices, Norms, Singular value decomposition (SVD).

QR Factorization and Least Squares: Projectors, QR factorization, Gram-Schmidt Orthogonalization, MATLAB, Householder triangularization, Least square problems. Conditioning and stability: Conditioning and condition numbers, Floating point arithmetic, Stability, Stability of householder triangularization, Stability of back substitution, Conditioning of least squares problems, Stability of least squares algorithm.

System of Equations: Gaussian Elimination, Pivoting, Stability of Gaussian Elimination,Cholesky Factorization.

Eigenvalues: Eigenvalue problems, Overview of eigenvalue algorithms, Reduction to Hessenberg or tridiagonal form, Rayleigh quotient, Inverse iteration, QR algorithms withoutshifts, QR algorithms with shifts, Computing the SVD.

Iterative methods: Overview of iterative methods, The Arnoldi iteration, The Lancoz iteration, Gauss quadrature, Conjugate gradients, Bi-orthogonalization methods, Preconditioning.

**Course Outcomes:** By the completion of this course the students would be able to:

- Use numerical linear algebra as building bricks in computation.
- Use computer algorithms, programs and software packages to compute solutions to currentproblems.
- Critically analyze and give advice regarding different choices of models, algorithms, andsoftware with respect to efficiency and reliability.
- Critically analyze the accuracy of the obtained numerical result and to present it in avisualized way.

**Text Book:** L. N. Trefethen and D. Bau, Numerical Linear Algebra, 1st Edition, Philadelphia, PA: Society for Industrial and Applied Mathematics, 1997. ISBN: 9780898713619.

**Reference Books:**

1. Bai et al., Templates for the Solution of Algebraic Eigenvalue Problems: A Practical Guide (Software, Environments and Tools), 1st Edition, Philadelphia, PA: Society for Industrial and Applied Mathematics, 2000. ISBN: 9780898714715.
2. Barret et al., Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods (Miscellaneous Titles in Applied Mathematics Series No 43), 1st Edition, Philadelphia, PA: Society for Industrial and Applied Mathematics, 1993. ISBN: 9780898713282

| **Weekly Breakdown** | | |
|---|---|---|
| ***Week*** | ***Section*** | ***Topics*** |
| 1 | 1.1, 1.2 | Matrix-vector multiplication, Orthogonal vectors and matrices |
| 2 | 1.3, 1.4 | Norms, Singular value decomposition (SVD) |
| 3 | 2.1-2.3 | Projectors, QR factorization, MATLAB |
| 4 | 2.4, 2.5 | Householder triangularization, Least square problems. |
| 5 | 3.1, 3.2 | Conditioning and condition numbers, Floating point arithmetic |
| 6 | 3.3-3.5 | Stability, Stability of householder triangularization, Stability of back substitution |
| 7 | 3.6, 3.7 | Conditioning of least squares problems, Stability of least squares algorithm |
| 8 | 4.1, 4.2 | Gaussian Elimination, Pivoting |
| 9 | **Mid Semester Exam** | |
| 10 | 4.3, 4.4 | Stability of Gaussian Elimination, Cholesky Factorization, |
| 11 | 5.1, 5.2 | Eigenvalue problems, Overview of eigenvalue algorithms |
| 12 | 5.3, 5.4 | Reduction to Hessenberg or tridiagonal form, Rayleigh quotient |
| 13 | 5.5-5.7 | Inverse iteration, QR algorithms without shifts, QR algorithms with shifts |

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| 14 | 5.8, 6.1, 6.2 | Computing the SVD, Overview of iterative methods, The Arnoldi iteration |
| 15 | 6.3-6.5 | The Lancoz iteration, Gauss quadrature, Conjugate gradients |
| 16 | 6.6, 6.7 | Bi-orthogonalization methods, Preconditioning. |
| 17 | | Review |
| 18 | **End Semester Exam** | |